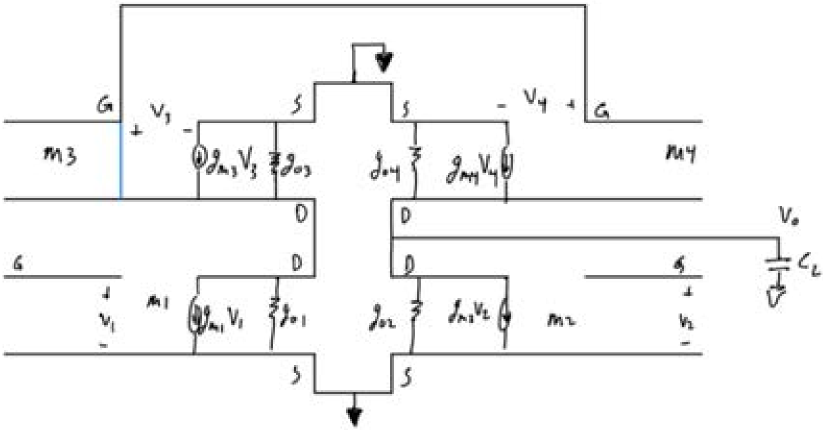
EE 435
Homework 2 Solutions
Spring 2024

Problem 1 and 2

Part A

Begin by drawing the amplifier's small-signal model.



Let $V_1 = V_{INP}$, $V_2 = V_{INN}$, and $V_4 = V_3$. We can form the following two equations:

$$(1): V_{OUT}(g_{o2} + g_{o4} + sC_L) + g_{m2}V_{INN} + g_{m4}V_3 = 0$$
$$(2): V_3(g_{o1} + g_{o3} + g_{m3}) + g_{m1}V_{INP} = 0$$

Part B

Rewrite (2) to allow for V_3 to be substituted:

$$(2.1): V_3 = \frac{-g_{m1}V_{INP}}{g_{o1} + g_{o3} + g_{m3}}$$

Substitute (2.1) into (1)

$$(1.2): V_{OUT}(g_{o2} + g_{o4} + sC_L) + g_{m2}V_{INN} + g_{m4}\frac{-g_{m1}V_{INP}}{g_{o1} + g_{o3} + g_{m3}} = 0$$

Solve for V_{OUT}:

$$(1.3): V_{OUT} = -\frac{g_{m2}V_{INN}}{g_{o2} + g_{o4} + sC_L} + \frac{g_{m4}g_{m1}V_{INP}}{(g_{o1} + g_{o3} + g_{m3})(g_{o2} + g_{o4} + sC_L)}$$
$$= \frac{-g_{m2}V_{INN} + \dfrac{g_{m4}g_{m1}V_{INP}}{g_{o1} + g_{o3} + g_{m3}}}{g_{o2} + g_{o4} + sC_L}$$

Part C

To make MATLAB solve for $A_V = \frac{V_{OUT}}{V_{IN}}$, it is necessary to put V_{INP} and V_{INN} in terms of the differential voltage: $V_{INP} = \frac{V_D}{2}$ and $V_{INN} = -\frac{V_D}{2}$. Note that, as soon as this is done, the math no longer correctly considers the effects of common-mode voltage.

```
% Create variables which will occur in the math
syms Vout VD gm1 gm2 gm3 gm4 go1 go2 go3 go4;
syms sCL V3;

% Create the first equation, equivalent to (1) from
%   the previous part
eqn1 = Vout * (go2 + go4 + sCL) - gm2 * VD/2 + gm4 * V3 == 0;

% Create the second equation, equivalent to (2) from
%   the previous part
eqn2 = V3 * (go1 + go3 + gm3) + gm1 * VD/2 == 0;

% To solve, convert both equations into a matrix.
[A, B] = equationsToMatrix([eqn1 eqn2], [Vout V3]);

% Solve the matrix
results = linsolve(A, B)
```

Formatted, the result is as follows:

$$V_{OUT} = \frac{V_D(g_{m1}g_{m4} + g_{m2}g_{m3} + g_{m2}g_{o1} + g_{m2}g_{o3})}{2(g_{m3} + g_{o1} + g_{o3})(g_{o2} + g_{o4} + sC_L)}$$

Equivalently:

$$\frac{V_{OUT}}{V_D} = A_V = \frac{(g_{m1}g_{m4} + g_{m2}g_{m3} + g_{m2}g_{o1} + g_{m2}g_{o3})}{2(g_{m3} + g_{o1} + g_{o3})(g_{o2} + g_{o4} + sC_L)}$$

Part D

$$A_V \approx \frac{(g_{m1}g_{m4} + g_{m2}g_{m3})}{2(g_{m3})(g_{o2} + g_{o4} + sC_L)}$$

Because (M_1, M_2) and (M_3, M_4) are matched, we can make the statement that $g_{m1} = g_{m2}$, $g_{o1} = g_{o2}$, $g_{m3} = g_{m4}$, and $g_{o3} = g_{o4}$. This allows us to simplify further:

$$A_V \approx \frac{2g_{m1}g_{m3}}{2g_{m3}(g_{o1} + g_{o3} + sC_L)} = \frac{g_{m1}}{g_{o1} + g_{o3} + sC_L}$$

Problem 3

Recall that a trans-resistance amplifier is an amplifier whose output voltage is equal to the applied input current multiplied by some gain, R_T. We can start by finding the input current, which is simple to do since an ideal trans-resistance amplifier has zero input impedance:

$$I_{IN} = \frac{V_{IN}}{R_1} + \frac{V_{OUT}}{R_2}$$

We know that the output voltage of the amplifier is given as follows:

$$V_{OUT} = R_T I_{IN}$$

We can adjust the two equations slightly to give us one equation in terms of V_{OUT} and V_{IN}:

$$\frac{V_{OUT}}{R_T} = \frac{V_{IN}}{R_1} + \frac{V_{OUT}}{R_2} \rightarrow V_{OUT}\left(\frac{1}{R_T} - \frac{1}{R_2}\right) = \frac{V_{IN}}{R_1} \rightarrow V_{OUT}\left(\frac{R_2 - R_T}{R_2 R_T}\right) = \frac{V_{IN}}{R_1}$$

$$\frac{V_{OUT}}{V_{IN}} = A_V = \frac{R_2 R_T}{R_1(R_2 - R_T)}$$

The voltage gain is thus $\frac{R_2 R_T}{R_1(R_2 - R_T)}$. As R_T approaches infinity, this simply becomes $-R_2/R_1$.

$$\lim_{R_T \to \infty} \frac{R_2 R_T}{R_1(R_2 - R_T)} = -\frac{R_2}{R_1}$$

Problem 4

Note: Solution written for ON 0.5u process as this is the process that students used for this HW. In this process, $\mu_n C_{OX} \approx 112.8\mu$ and $V_{Tn} \approx 0.79V$.

Part A and B

If $V_{INQ} = 0.5V$, $V_{OUTQ} = 0.5V$, $V_{DD} = 1V$, and $V_{SS} = -1V$, then the transistor's quiescent V_{DS} and V_{GS} voltages are both $1.5V$. Recall that, for a transistor to be saturated, V_{DS} must be greater than or equal to $V_{GS} - V_T$. If V_{DS} and V_{GS} are equal, this will be true so long as the MOSFET is on ($V_{GS} > V_T$). So, we can assume the transistor is saturated.

Now, recall the equation for a MOSFET placed in saturation. Note the inclusion of the $1 + \lambda V_{DS}$ term.

$$I_D = \frac{1}{2}\mu_n C_{OX} \frac{W}{L}(V_{GS} - V_{Tn})^2(1 + \lambda V_{DS})$$

What is λ? Recall from EE 330 that λ is highly dependent on device length, increasing dramatically with small lengths. It is also dependent on the current through the device; in EE 330, it was observed that a higher current increased λ. Using methods similar to the ones used in EE 330, an approximate value of λ can be obtained. A reasonable value obtained for a $1.8\mu m$ length transistor is $0.02055V^{-1}$.

Substituting in known values, we can solve for W/L:

$$4mA = \frac{1}{2}(112.8\mu)\left(\frac{W}{L}\right)(1.5 - 0.79)^2(1 + 0.02055 * 1.5)$$

$$\frac{W}{L} = 136.483$$

This W/L ratio can be confirmed in Virtuoso by recreating the circuit and obtaining the circuit's DC operating point. Doing this yields a V_{OUT} of approximately $-0.1186V$ instead of $0.5V$. This is the result of error (see the image below, which illustrates how dramatically V_{OUT} changes with the W/L ratio in this circuit).

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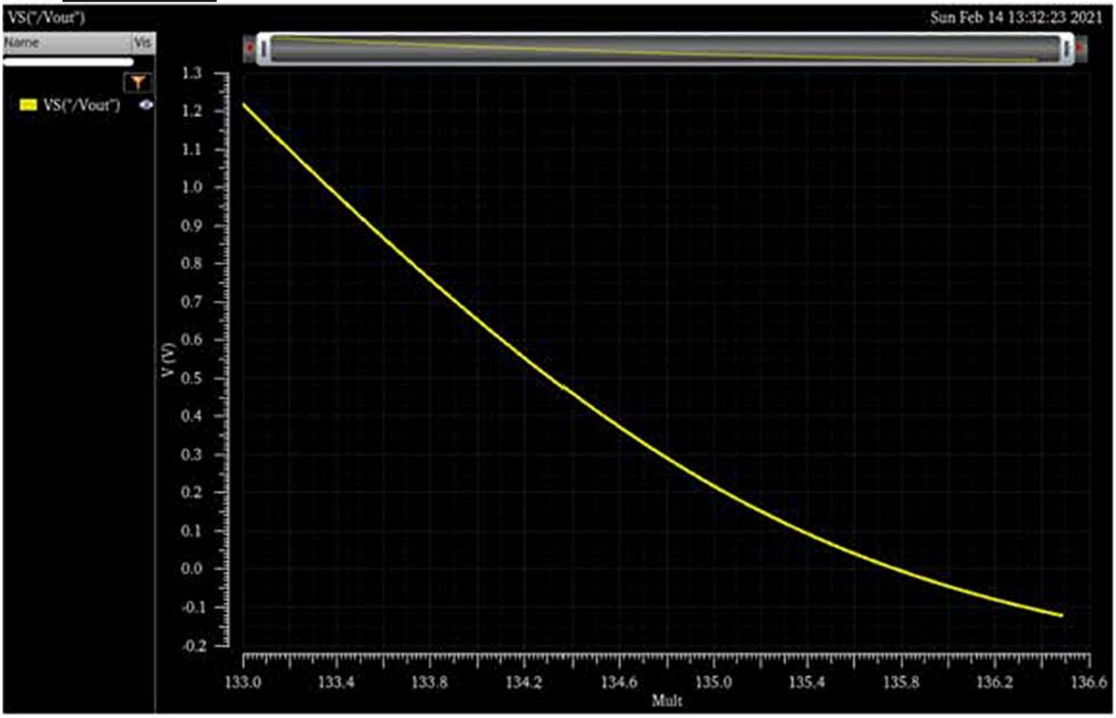

Adjusting our work slightly shows that a width-length ratio of 134.3 yields a quiescent output voltage of $507mV$, which is well within 2.5% of the desired $0.5V$ point.

Part C

Voltage gain can be expressed as $A_V = \frac{g_m}{g_o}$. g_m and g_o can be calculated as follows:

$$g_m = \sqrt{2\mu_n C_{OX}\frac{W}{L}I_{DQ}} = \sqrt{2 * 112.8\mu * 134.3 * 4m} \approx 11m$$

$$g_o \approx \lambda I_{DQ} = 0.02055 * 4m = 82.2\mu$$

Thus:

$$A_V \approx 133.9$$

Part D

To confirm the amplifier's small-signal gain, apply a small-signal AC input and observe the amplifier's small-signal output. Dividing the input and output magnitude yields the small-signal gain. Doing so yields a gain of approximately 139.5. This is within 5% of the expected gain.

Problem 5 and 6

Recall the following equations for the design of a ST amplifier:

$$A_0 = \frac{1}{\lambda_1 + \lambda_3}\frac{1}{V_{EB1}}$$

$$\omega_{GBW} = \frac{P}{V_{DD}C_L}\frac{1}{2V_{EB1}}$$

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One design process is to choose the length of the NMOS and PMOS transistors such that we know λ_1 and λ_3, and to then solve for what V_{EB1} is needed to obtain the gain. Then, we can solve for the current which must be drawn through M_{1-4} to obtain the desired GBW. All remaining excess bias voltages can be selected for convenience of calculations.

An Excel table is useful for designing op-amps:

Key

Color	Def.
	Calculated / Equation
	Constraint
	DoF
	Final Result

Constraints

Item	Value	Unit
VDD	3.5E+0	V
VSS	000.0E+0	V
CL	1.0E-12	F
GBW	2.0E+6	Hz
	12.6E+6	Rad
A0	50.0E+0	dB
	316.2E+0	V/V

Process Parameters

Item	Value	Unit
UnCox	112.8E-6	A/V^2
Un/Up	3.0E+0	
UpCox	37.6E-6	A/V^2
Vtn	790.0E-3	V
Vtp	920.0E-3	V

Design

Name	Value	Description	
L1	5.0E-6	Length of M1	
M1 Lambda	19.2E-3	Lambda 1	
L2	5.0E-6	Length of M2	
M2 Lambda	19.2E-3	Lambda 2	
L3	5.0E-6	Length of M3	
M3 Lambda	19.7E-3	Lambda 3	
L4	5.0E-6	Length of M4	
M4 Lambda	19.7E-3	Lambda 4	
L5	5.0E-6	Length of M5	
M5 Lambda	19.2E-3	Lambda 5	
VEB1	158.1E-3		
VEB2	158.1E-3		
VEB3	200.0E-3		
VEB4	200.0E-3		
VEB5	200.0E-3		
P	13.9E-6		
I	4.0E-6		
M1	1.4E+0	W1	7.0E-6
M2	1.4E+0	W2	7.0E-6
M3	2.6E+0	W3	13.2E-6
M4	2.6E+0	W4	13.2E-6
M5	1.8E+0	W5	8.8E-6
VB1	2.4E+0		11.9E-6
VB2	990.0E-3		

Simulating the amplifier gives a gain of $42.5dB$ and a gain-bandwidth of $1.759MHz$. This is below spec but is improved by decreasing V_{EB1}, λ_1, and λ_3 further.

Problem 8

Part A

For the first amplifier:

Write down what we know first:

$$V_{OUT} = A(s)[V_{IN} - V_1]$$
$$V_1 = \frac{R_1 V_{OUT}}{R_2 + R_1}$$

Now, substitute in:

$$V_{OUT} = A(s)\left[V_{IN} - \frac{R_1 V_{OUT}}{R_2 + R_1}\right]$$

Divide each side to find closed-loop gain:

$$\frac{V_{OUT}}{V_{IN}} = A_{CL} = A(s)\left[1 - \frac{R_1 A_{CL}}{R_2 + R_1}\right]$$

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$$A_{CL}\left[1+\frac{R_1}{R_2+R_1}A(s)\right]=A(s)$$

$$A_{CL}=\frac{A(s)}{1+\frac{R_1}{R_2+R_1}A(s)}=\frac{\frac{GB}{s}}{1+\frac{R_1}{R_2+R_1}\frac{GB}{s}}=\frac{GB}{s+\frac{R_1}{R_2+R_1}GB}=\frac{\frac{R_2+R_1}{R_1}}{\frac{s}{\frac{R_1}{R_2+R_1}GB}+1}$$

For the second amplifier:

Write down what we know first:

$$V_{OUT}=A(s)[-V_1]$$

$$\frac{V_1-V_{IN}}{R_1}+\frac{V_1-V_{OUT}}{R_2}=0$$

Now, solve for V_1 and substitute:

$$\frac{-\frac{V_{OUT}}{A(s)}-V_{IN}}{R_1}+\frac{-\frac{V_{OUT}}{A(s)}-V_{OUT}}{R_2}=0$$

$$-R_2\left[\frac{V_{OUT}}{A(s)}+V_{IN}\right]-R_1\left[\frac{V_{OUT}}{A(s)}+V_{OUT}\right]=0$$

$$-V_{OUT}\left[\frac{R_2}{A(s)}+\frac{R_1}{A(s)}+R_1\right]-V_{IN}[R_2]=0$$

$$\frac{V_{OUT}}{V_{IN}}=A_{CL}=-\frac{R_2}{\frac{R_2}{A(s)}+\frac{R_1}{A(s)}+R_1}=-\frac{R_2A(s)}{R_2+R_1+R_1A(s)}=-\frac{\frac{R_2GB}{s}}{R_2+R_1+\frac{R_1GB}{s}}$$

$$A_{CL}=-\frac{R_2GB}{s(R_2+R_1)+R_1GB}=-\frac{R_2/R_1}{\frac{s}{\frac{R_1GB}{R_2+R_1}}+1}$$

Part B

In the forms the solutions were left in in Part A, the 3dB bandwidth is simply the denominator of the s/x fraction. Thus, for both circuits, the 3dB bandwidth (in radians/second) is $\frac{R_1}{R_1+R_2}GB$.

Problem 9 It was stated in class that all even-ordered distortion terms introduced by the amplifier vanish in symmetric fully differential amplifiers. Prove this fact. (Hint: assume that if an ideal differential sinusoidal signal is applied at the input, one of the single-input outputs is given by the expression

$$V_{OUT1}(t) = A_1 \sin(\omega_1 t + \theta_1) + \sum_{k=2}^{\infty} A_k \sin(k\omega_1 t + \theta_k)$$

where ω_1 is the frequency of the sinusoidal input and the parameters A_2, A_3, ... and θ_2, θ_3, ... characterize the distortion introduced by the amplifier.)

Solution: The "Hint" may not be helpful. Denote V_{O1} and V_{O2} as the two outputs of the symmetric fully differential amplifier. If the input on one side is $V_d/2$ and on the other side it is $-V_d/2$, it follows from symmetry that

$$V_{OUT1} = \sum_{i=1}^{\infty} A_i V_d^i$$

$$V_{OUT2} = \sum_{i=1}^{\infty} A_i \left(-V_d\right)^i$$

Thus the differential output is given by

$$V_{OUTDIFF} = V_{OUT1} - V_{OUT2} = \sum_{i=1}^{\infty} A_i \left(V_d^i - \left(-V_d\right)^i\right)$$

But

$$\sum_{i=1}^{\infty} A_i \left(V_d^i - \left(-V_d\right)^i\right) = \sum_{i=1}^{\infty} A_i \left(V_d^i - \left(-1\right)^i V_d^i\right)$$

For i even, the terms in () vanish and for i odd, the terms in the () are of the same sign so

$$V_{OUTDIFF} = \sum_{\substack{i=1 \\ i\ \text{odd}}}^{\infty} 2A_i V_d^i$$

So now assume $V_d = V_m \sin(\omega t)$. Thus

$$V_{OUTDIFF} = \sum_{\substack{i=1 \\ i\ \text{odd}}}^{\infty} 2A_i \sin^i(\omega t)$$

Observe that $V_{OUTDIFF}$ is comprised only of odd powers of sin(ωt). But from trig identities, odd powers of sin(ωt) include only odd harmonic terms and thus no even-ordered distortion terms are present.